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FORCED AXI-SYMMETRIC RESPONSE OF POLAR ORTHOTROPIC LINEARLY TAPERED CIRCULAR PLATES

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Forced axi-symmetric vibrations of polar orthotropic linearly tapered circular plates are discussed on the basis of classical plate theory. Ritz method has been employed to obtain the solutions. The deflection function and the bending moments for forced vibrations of the plate are presented for various values of taper parameter and rigidity ratio. A comparison of results with those available in literature shows an excellent agreement.

1. INTRODUCTION

Circular plates of variable thickness are widely used in various engineering structures and machines. The study of dynamic response of these plates derived from polar orthotropic theory is of great practical importance due to the increasing use of plates stiffened in radial and circumferential directions and plates fabricated out of modern composites. The consideration of thickness variation together with orthotropy meets the desirability of light weight along with high specific strength and stiffness.

Although free vibration problem of polar orthotropic circular and annular plates of variable thickness has been studied by a number of research workers [1–9], the authors have not come across any paper dealing with forced vibration of polar orthotropic plates of variable thickness. The study of dynamic response of circular plates is essential due to their use in design of machine parts such as diaphragm of turbines, pistons of engines and transducers.

Forced vibration analysis of isotropic, circular, rectangular, and annular plates of uniform thickness has been presented by Reismann [10, 11], Mcleod and Bishop [12], Donaldson [13], Laura and Duran [14] and Laura *et al.* [15–18] etc. Laura *et al.* [19] studied the dynamic behaviour of isotropic circular plates with stepped thickness and elastically restrained edge. Forced axisymmetric response of isotropic circular plate has also been studied by Chandrasekharan Kanukkasseril [20], while Beaudan and Reismann [21] analyzed forced flexural motion of isotropic uniform rectangular plates taking into account viscous damping. Leissa [22–24] and Leissa and Chern [25] analysed forced vibration of plates with and without damping. Some recent researches on forced vibration of isotropic plates are presented by Weisensel and Schlack [26, 27]. Most recently, Gupta and Goel [28, 29] have studied forced axisymmetric and asymmetric vibrations of linearly tapered isotropic circular plates.

The present paper analyses forced axisymmetric response of polar orthotropic circular plates of linearly varying thickness with elastically restrained edge using Ritz method. The plate is subjected to a $P(r)\cos\omega t$ type excitation for three types of loadings (i) when load is distributed uniformly over the entire plate, (ii) when load is distributed uniformly on the

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annular region extending from r_0 to r_1, and (iii) when load is distributed uniformly on the disk extending from 0 to r_1, the total load on the plate being constant for all the cases.

Numerical results for amplitudes of displacement at the centre are obtained for all the three types of loadings for different values of taper parameter, rigidity ratio and flexibility parameter. Results are also presented graphically for transverse deflection, radial and tangential bending moments for different values of plate parameters. Results for radial and tangential bending moments can be obtained for circumferentially stiffened plates ($E_\theta > E_r$), but not for radially stiffened plates ($E_\theta < E_r$) because in this case infinite stress is developed at the centre ([1], p. 372). The transverse deflection and bending moments for clamped and simply supported isotropic plates of uniform thickness are obtained as special cases, which on comparison with published results are found to match exactly.

2. ENERGY EXPRESSION

Consider a thin circular plate of radius a, thickness $h = h(r)$, elastically restrained against rotation and subjected to $P(r)\cos\omega t$ type of excitation extending from $r = r_0$ to $r = r_1$. Let (r, θ) be the polar co-ordinates of any point on neutral surface of the plate referred to the centre of the plate as origin (Figure 1).

The maximum kinetic energy of the plate is given by

$$T_{max} = \frac{1}{2}\rho\omega^2 \int_0^a \int_0^{2\pi} h w^2 r \, \mathrm{d}\theta \, \mathrm{d}r \tag{1}$$

where w is the transverse deflection, ρ the mass density, and ω the frequency in radians per second. The maximum strain energy of the plate is given by

$$U_{max} = \frac{1}{2}\int_0^a \int_0^{2\pi} \left[D_r \left\{ \left(\frac{\partial^2 w}{\partial r^2}\right)^2 + 2\nu_\theta \frac{\partial^2 w}{\partial r^2}\left(\frac{1}{r}\frac{\partial w}{\partial r}\right) \right\} + D_\theta \left(\frac{1}{r}\frac{\partial w}{\partial r}\right)^2 \right] r \, \mathrm{d}r \, \mathrm{d}\theta$$
$$+ \frac{1}{2} a k_\phi \int_0^{2\pi} \left(\frac{\partial w(a, \theta)}{\partial r}\right)^2 \mathrm{d}\theta, \tag{2}$$

where $1/k_\phi$ is the rotational flexibility of the springs and $D_r(r) = E_r h^3/12(1 - \nu_\theta \nu_r)$, $D_\theta(r) = E_\theta h^3/12(1 - \nu_\theta \nu_r)$ are the flexural rigidities of the plate.

The work done by external force $P(r)$ acting on the plate in the direction parallel to z-axis is given by

$$V_{max} = \int_0^{2\pi} \mathrm{d}\theta \int_{r_0}^{r_1} w P(r) r \, \mathrm{d}r. \tag{3}$$

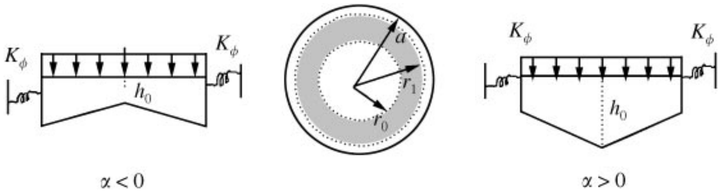

Figure 1.

3. METHOD OF SOLUTION: RITZ METHOD

Ritz method requires that the functional

$$J(w) = U_{max} - V_{max} - T_{max} = \frac{1}{2}\int_0^a \int_0^{2\pi} \left[D_r \left\{ \left(\frac{\partial^2 w}{\partial r^2} \right)^2 + 2v_\theta \frac{\partial^2 w}{\partial r^2} \left(\frac{1}{r}\frac{\partial w}{\partial r} \right) \right\} + D_\theta \left(\frac{1}{r}\frac{\partial w}{\partial r} \right)^2 \right] r\,\mathrm{d}r\,\mathrm{d}\theta$$

$$+ \frac{1}{2} a k_\phi \int_0^{2\pi} \left(\frac{\partial w(a,\theta)}{\partial r} \right)^2 \mathrm{d}\theta - \int_0^{2\pi} \mathrm{d}\theta \int_{r_0}^{r_1} w P(r) r\,\mathrm{d}r - \frac{1}{2}\rho\omega^2 \int_0^a \int_0^{2\pi} h w^2 r\,\mathrm{d}\theta\,\mathrm{d}r \tag{4}$$

be minimized.

For a uniformly distributed load P_0 extending from $r = r_0$ to $r = r_1$

$$P(r) = q_0 = \frac{P_0}{\pi(r_1^2 - r_0^2)}[U(r - r_0) - U(r - r_1)].$$

Introduce the non-dimensional variables $W = w/(a^4 q_0/D_{r0})$, $R = r/a$, $R_1 = r_1/a$, $R_2 = r_2/a$ and assume the deflection function as

$$W(R) = \sum_0^m A_i W_i(R) = \sum_1^m A_i(1 + \gamma_i R^4 + \beta_i R^{1+p}) R^{2(i-1)}, \tag{5}$$

where A_i are undetermined coefficients, $p^2 = E_\theta/E_r$ and γ_i, β_i are unknown constants to be determined from the boundary conditions

$$K_\phi \frac{\mathrm{d}W_i}{\mathrm{d}R}\bigg|_{R=1} = -(1-\alpha)^3 \left[\frac{\mathrm{d}^2 W_i}{\mathrm{d}R^2} + v_\theta \left(\frac{1}{R}\frac{\mathrm{d}W_i}{\mathrm{d}R} \right) \right]_{R=1}, \tag{6}$$

$$W_i(1) = 0 \tag{7}$$

(Leissa [22], p. 14), given by

$$\gamma_i = \frac{S_i - U_i}{Q_i - S_i}, \quad \beta_i = \frac{U_i - Q_i}{Q_i - S_i},$$

where

$$Q_i = K_\phi(2i + n + 2) + (1-\alpha)^3\{(2i + n + 2)(2i + n + 1) + v_\theta(2i + n + 2 - n^2)\},$$

$$S_i = K_\phi(2i + n + p - 1) + (1-\alpha)^3\{(2i + n + p - 1)(2i + n + p - 2) + v_\theta(2i + n + p - 1 - n^2)\},$$

$$U_i = K_\phi(2i + n - 2) + (1-\alpha)^3\{(2i + n - 2)(2i + n - 3) + v_\theta(2i + n - 2 - n^2)\}.$$

The choice of functions approximating the deflection function in equation (5) is based upon the static deflection of polar orthotropic circular plates [1, p. 372], which has faster rate of convergence [7] as compared to polynomial co-ordinate functions [15–19]. The functional $J(w)$ given by (4) on introduction of non-dimensional variables and using relation (5) becomes

$$J(W) = \pi D_{r0}\left[\int_0^1\left[(1-\alpha R)^3\left\{\left(\frac{\mathrm{d}^2W}{\mathrm{d}R^2}\right)^2 + 2v_\theta\frac{\mathrm{d}^2W}{\mathrm{d}R^2}\left(\frac{1}{R}\frac{\mathrm{d}W}{\mathrm{d}R}\right)\right\} + p^2\left(\frac{1}{R}\frac{\mathrm{d}W}{\mathrm{d}R}\right)^2\right]R\,\mathrm{d}R\right.$$

$$\left. + K_\phi\left(\frac{\mathrm{d}W}{\mathrm{d}R}\right)^2_{R=1} - 2\int_{R_1}^{R_2} w(R)R\,\mathrm{d}R - \Omega^2\int_0^1(1-\alpha R)W^2R\,\mathrm{d}R\right], \tag{8}$$

where $h = h_0(1-\alpha R)$ specifies the linear thickness variations, h_0 being the thickness of the plate at the centre, α the taper parameter and $D_{r0} = E_r h_0^3/12(1-v_\theta v_r)$, $\Omega^2 = a^4\omega^2\rho h_0/D_{r0}$, $K_\phi = ak_\phi/D_{r0}$.

The minimization of the functional $J(W)$ given by equation (8) requires

$$\frac{\partial J(W)}{\partial A_i} = 0, \quad i = 1, \ldots, m. \tag{9}$$

This leads to a system of non-homogeneous equations in A_j,

$$(a_{ij} - \Omega^2 b_{ij})A_j = C_i, \quad i, j = 1, \ldots, m, \tag{10}$$

where

$$a_{ij} = \int_0^1 (1-\alpha R)^3\left[W_i''W_j'' + 2v_\theta W_i''\left(\frac{W_j'}{R}\right) + p^2\left(\frac{W_i'}{R}\right)\left(\frac{W_j'}{R}\right)\right]RdR + K_\Phi W_i'(1)W_j'(1), \tag{11}$$

$$b_{ij} = \int_0^1 (1-\alpha R)W_iW_jR\,\mathrm{d}R \tag{12}$$

and

$$C_i = 2\int_0^1 W_iR\,\mathrm{d}R. \tag{13}$$

The solution of the system of equations (10) gives the values of A_i and hence the transverse deflection W and the radial and tangential bending moments

$$\frac{M_r}{q_0a^2} = -(1-\alpha R)^3\left[\frac{\mathrm{d}^2W}{\mathrm{d}R^2} + v_\theta\frac{1}{R}\frac{\mathrm{d}W}{\mathrm{d}R}\right], \tag{14}$$

$$\frac{M_\theta}{a^2q_0} = (1-\alpha R)^3\left[v_\theta\frac{\mathrm{d}^2W}{\mathrm{d}R^2} + p^2\frac{1}{R}\frac{\mathrm{d}W}{\mathrm{d}R}\right] \tag{15}$$

are computed.

4. NUMERICAL RESULTS

Numerical results have been calculated for forced vibration of polar orthotropic plates for various values of taper parameter α (= 0; $\pm$0·3), rigidity ratio E_θ/E_r (= 1·00, 2·00, 5·00) and flexibility parameter K_ϕ(= 0, 10, $10^{20} \cong \infty$). The natural frequencies for free vibrations are obtained by putting $P(r) = 0$. In case of forced vibration the non-dimensional frequency parameter is taken as $\Omega = \eta\Omega_{00}$ for $\Omega < \Omega_{00}$ and $\Omega = \Omega_{00} + \eta(\Omega_{01} - \Omega_{00})$ for

$\Omega_{00} < \Omega < \Omega_{01}$. The normalized deflection and bending moments are obtained for $\eta = 0{\cdot}2$, Poisson's ratio ν_θ of the plate being fixed as 0·3.

5. DISCUSSION

In case of forced vibration problems the deflection and bending moments are of great interest to know from the design point of view. The results are presented graphically for transverse deflection and bending moments (radial as well as tangential) for various values of plate parameters i.e., $E_0/E_r(= 1{\cdot}0,\ 5{\cdot}0)$, $\alpha(= 0; \pm 0{\cdot}3)$, $K_\phi(= 0{\cdot}0,\ 10,\ 10^{20} \cong \infty)$ and $\eta = 0{\cdot}2$. The radial and tangential bending moments at the centre of the plate ($R = 0$) are zero for orthotropic plates, whereas they are non zero in case of isotropic plates. For other values of radial co-ordinate R, the bending moments are found to be dependent on the orthotropic nature of the plate, i.e., whether the plate is radially stiffened or circumferentially stiffened. Numerical results for radial bending moment M_r/q_0a^2 and tangential bending moment M_θ/q_0a^2 are given for $E_\theta > E_r$, i.e., only for circumferentially stiffened plates and not for radially stiffened plates, i.e., $E_\theta < E_r$, because in this case infinite stress is developed at the centre [28, p. 372]. The function assumed here to approximate transverse deflection is based upon the static deflection for polar orthotropic circular plates [28, p. 370]. The results for deflection and bending moments are presented for orthotropy parameter E_θ/E_r (= 5·0), taper parameter α (= 0; ±0·3) and flexibility parameter K_ϕ (= 0, 10, $10^{20} \cong \infty$) for various types of loadings i.e.,

(a) when load is distributed uniformly over the entire plate, presented in Figures 2–4;
(b) when load is distributed uniformly on the annular region extending from $r_0 = 0{\cdot}3$ to $r_1 = 0{\cdot}7$, presented in Figures 5–7;

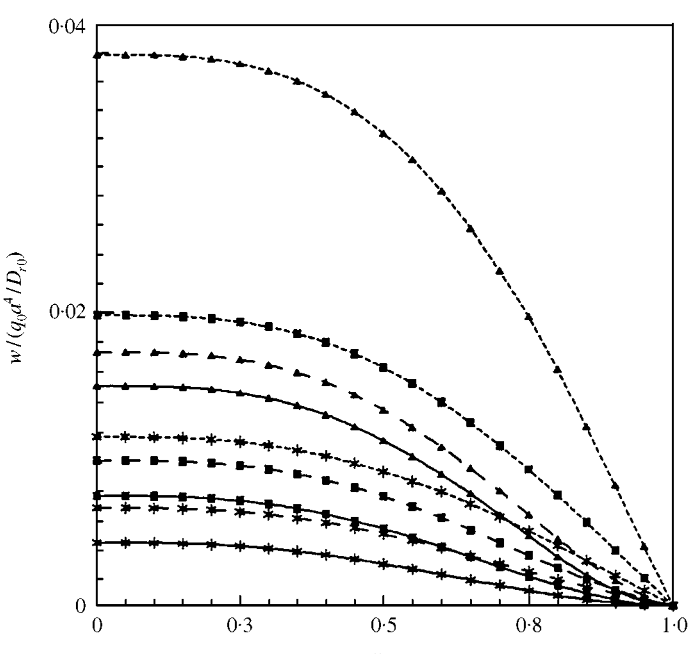

Figure 2. Deflection versus R. *****, $\alpha = -0{\cdot}3$; ■■■■■, $\alpha = 0{\cdot}0$; ▲▲▲▲▲, $\alpha = 0{\cdot}3$.

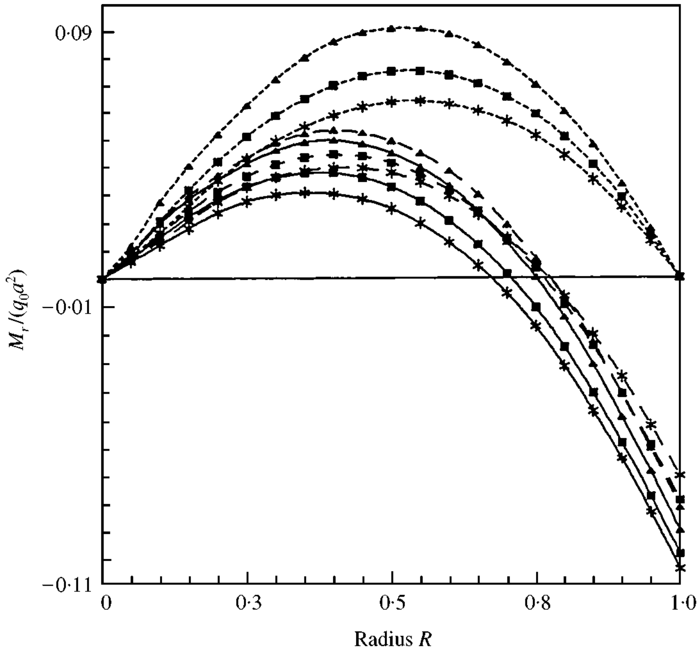

Figure 3. Radial bending moment versus R. -------, $K_0 = 0$ (SS); - - - -, $K_0 = 10$ and ——, $K_0 = 10^{20}$ (Cl); *****, $\alpha = -0{\cdot}3$; ■■■■■, $\alpha = 0{\cdot}0$; ▲▲▲▲▲, $\alpha = 0{\cdot}3$.

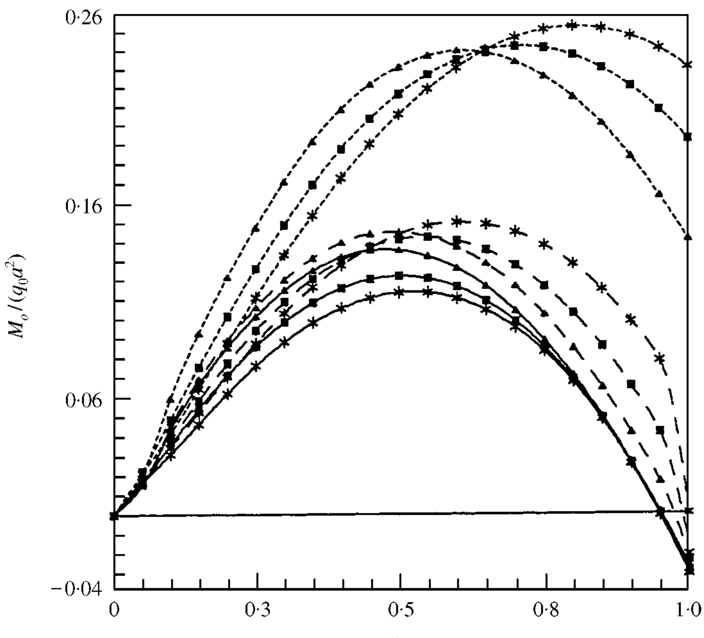

Figure 4. Tangential bending moment versus R. *****, $\alpha = -0{\cdot}3$; ■■■■■, $\alpha = 0{\cdot}0$; ▲▲▲▲▲, $\alpha = 0{\cdot}3$.

(c) when load is distributed uniformly on the disk extending from $r_0 = 0$ to $r_1 = 0{\cdot}5$, presented in Figures 8–10;

the total load on the plate being constant for all the cases.

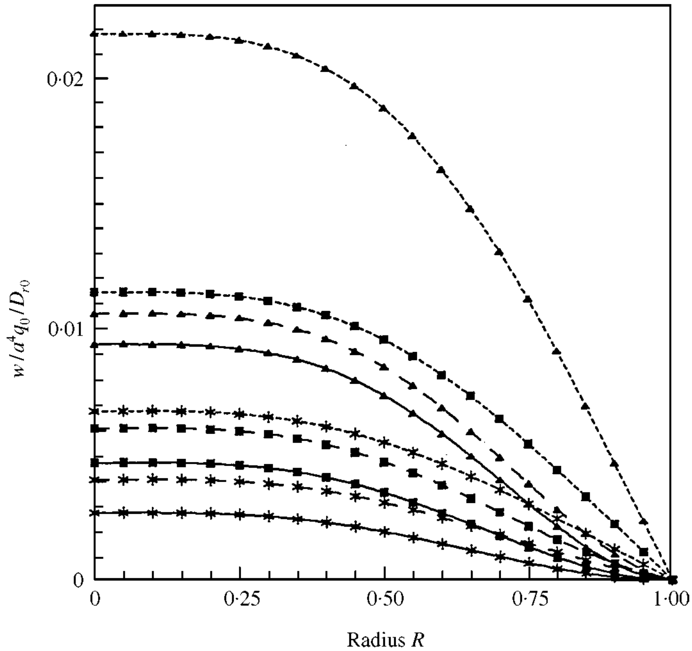

Figure 5. Deflection versus R. *****, $\alpha = -0{\cdot}3$; ■■■■■, $\alpha = 0{\cdot}0$; ▲▲▲▲▲, $\alpha = 0{\cdot}3$.

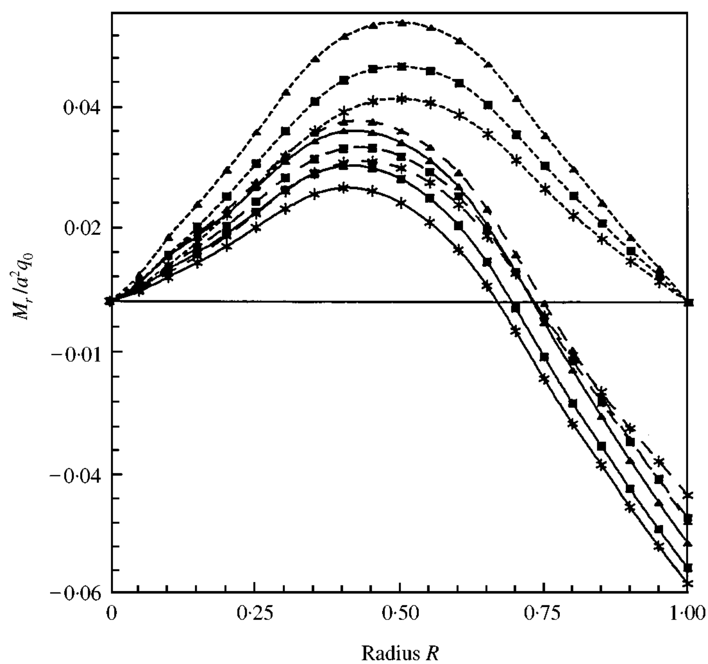

Figure 6. Radial bending moment versus R. *****, $\alpha = -0{\cdot}3$; ■■■■■, $\alpha = 0{\cdot}0$; ▲▲▲▲▲, $\alpha = 0{\cdot}3$.

Figures 2–5 present the deflection function and bending moments (radial as well as tangential) for simply supported edge (SS, i.e., $K_\phi = 0$), K_ϕ ($= 10$) and clamped edge (C1, i.e., $K_\phi = 10^{20} \cong \infty$) conditions. Figure 2 presents the deflection $W/(a^4q_0/D_{r0})$ at various points of the plate along radius for taper parameter α ($= 0$; $\pm$ 0·3). The deflection is maximum at

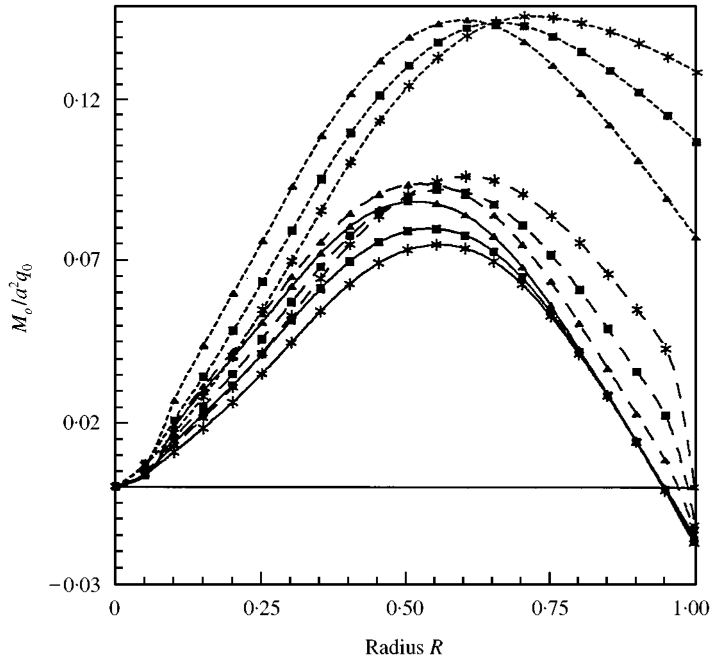

Figure 7. Tangential bending moment versus R. *****, $\alpha = -0{\cdot}3$; ■■■■■, $\alpha = 0{\cdot}0$; ▲▲▲▲▲, $\alpha = 0{\cdot}3$.

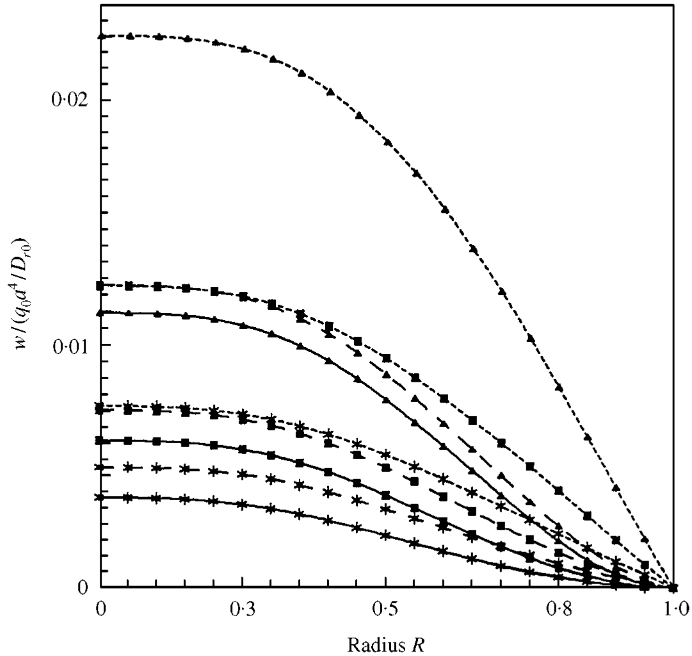

Figure 8. Deflection versus R. *****, $\alpha = -0{\cdot}3$; ■■■■■, $\alpha = 0{\cdot}0$; ▲▲▲▲▲, $\alpha = 0{\cdot}3$.

the centre for all the three types of loading irrespective of plate parameters. Further, the deflection for a simply supported plate is found to be greater than that for a clamped edge plate having other plate parameters fixed. The figure also demonstrates that the deflection for a uniform plate is greater than a centrally thinner plate but less than that for centrally

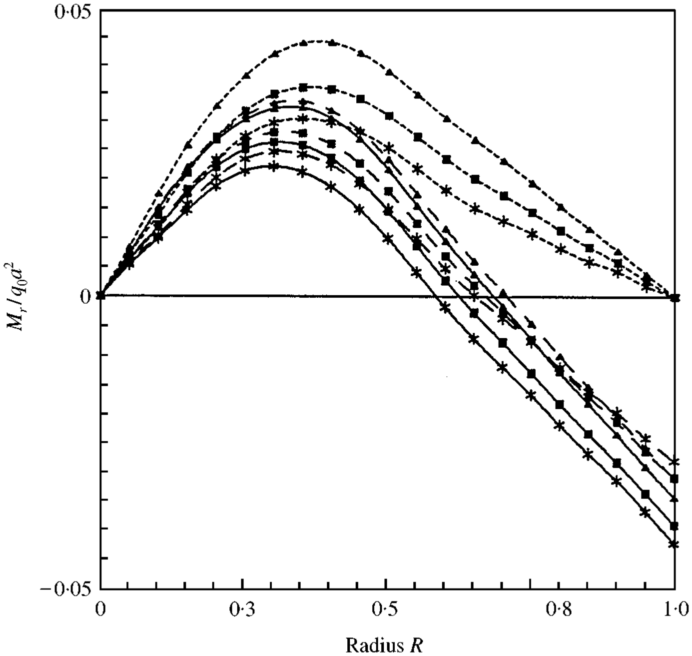

Figure 9. Radial bending moment versus R. *****, $\alpha = -0{\cdot}3$; ■■■■■, $\alpha = 0{\cdot}0$; ▲▲▲▲▲, $\alpha = 0{\cdot}3$.

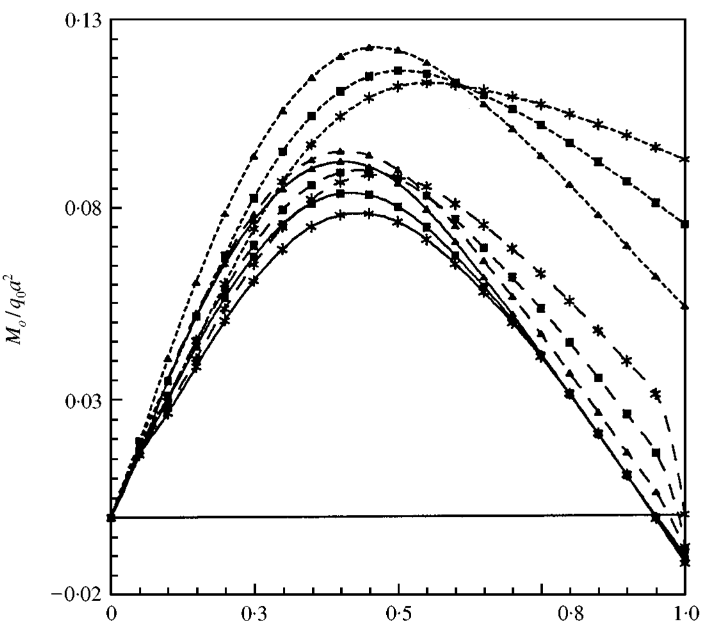

Figure 10. Tangential bending moment versus R. *****, $\alpha = -0{\cdot}3$; ■■■■■, $\alpha = 0{\cdot}0$; ▲▲▲▲▲, $\alpha = 0{\cdot}3$.

thicker plate for all the edge conditions. The radial and tangential bending moments, M_r/q_0a^2 and M_θ/q_0a^2 are presented in Figures 3–4 respectively. The behaviour of tangential bending moment is similar to that for a radial bending moment except for $R > 0{\cdot}7$ in case of simply supported plate. Figures 5–7 represent deflection and radial as well as tangential

TABLE 1

The amplitude of dimensionless displacement $W/(a^4 q_0/D_{r0})$ at the centre as a function of rigidity ratio E_θ/E_r, taper parameter α, flexibility parameter K_ϕ and loadings

K_ϕ		Rigidity ratio $E_\theta/E_r = 2{\cdot}0$				Rigidity ratio $E_\theta/E_r = 5{\cdot}0$			
		$\Omega < \Omega_{00}$		$\Omega_{00} < \Omega < \Omega_{01}$		$\Omega < \Omega_{00}$		$\Omega_{00} < \Omega < \Omega_{01}$	
		$\alpha = -0{\cdot}3$	$\alpha = 0{\cdot}3$	$\alpha = -0{\cdot}3$	$\alpha = 0{\cdot}3$	$\alpha = -0{\cdot}3$	$\alpha = 0{\cdot}3$	$\alpha = -0{\cdot}3$	$\alpha = 0{\cdot}3$
0·0	Ω	1·4713	0·9881	13·5565	9·4119	2·1068	1·3173	17·6390	11·7519
	Disk	0·012606	0·032875	– 0·004395	– 0·010756	– 0·005661	0·017034	– 0·002533	– 0·006582
	Annular	0·014821	0·040901	– 0·006485	– 0·016054	0·006775	0·021834	– 0·003911	– 0·010205
	Uniform	0·025723	0·070918	– 0·011014	– 0·027873	0·011736	0·037899	– 0·006682	– 0·017904
10·0	Ω	2·2332	1·6247	17·5278	13·1683	3·3688	2·0996	21·3514	15·6403
	Disk	0·006617	0·015104	– 0·003837	– 0·008191	0·003752	0·009294	– 0·002336	– 0·005268
	Annular	0·006876	0·016334	– 0·005224	– 0·011052	0·004085	0·010603	– 0·003364	– 0·007487
	Uniform	0·011712	0·026827	– 0·008489	– 0·17600	0·006904	0·017309	– 0·005489	– 0·011995
10^{20}	Ω	2·8525	1·7415	21·6267	14·0486	3·4948	2·1065	25·9158	16·6680
	Disk	0·004758	0·013692	– 0·003257	– 0·007619	0·002822	0·008458	– 0·002033	– 0·004916
	Annular	0·004405	0·014378	– 0·003993	– 0·010017	0·002772	0·009388	– 0·002660	– 0·006823
	Uniform	0·007354	0·023316	– 0·006098	– 0·015556	0·004542	0·015080	– 0·004039	– 0·010623

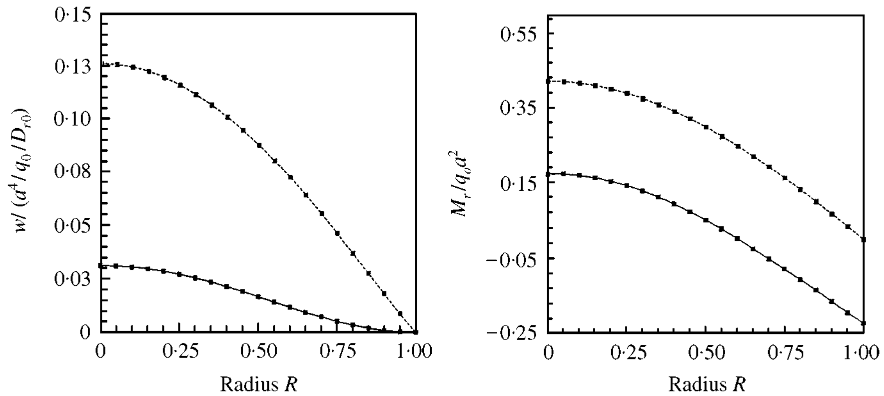

Figure 11. Deflection and bending moment for -----, $K_\phi = 0$ (SS); ——, $K_\phi = 10^{20}$ (Cl); ■■■■■ From reference [17].

TABLE 2

Comparison of displacement $W/(a^4 q_0/D_{r0})$ for simply supported circular plate of linearly varying thickness for $\nu_\theta = 0{\cdot}25$

α	Reference [17]	Present
– 0·5708	0·1817	0·1815
0·0	0·06568	0·0656
– 0·33333	0·1143	0·1127

bending moments for the ring loading. The deflection in this case is found to be small at each point as compared to that of uniform loading over the entire plate. The deflection in case of full plate loading is found to be more pronounced than that for disk or ring type loading, which can be seen from Figures 2, 5 and 8. The behaviour of radial and tangential bending moments for disk loading is observed to be greater than that for a ring loading but less than that for a uniformly loaded plate (Figures 3–4, 6–7 and 9 and 10).

Table 1 depicts the transverse deflection $W/(a^4 q_0/D_{r0})$ at the centre for different values of rigidity ratio E_θ/E_r, taper parameter α, flexibility parameter K_ϕ and different types of loadings. The table shows that the absolute value of transverse deflection for taper parameter $\alpha = -0{\cdot}3$ is less than that for taper parameter $\alpha = 0{\cdot}3$ for $\Omega < \Omega_{00}$ as well as $\Omega_{00} < \Omega < \Omega_{01}$. The transverse deflection decreases as the flexibility parameter K_ϕ increases. Further it can be observed that the transverse deflection decreases as the rigidity ratio E_θ/E_r increases, whatever be the other plate parameters.

A comparison of results for the deflection and bending moments for simply supported and clamped edge conditions obtained by Laura [17] using Galerkin's method for uniformly loaded plate of constant thickness is presented in Figure 11 for $\Omega/\Omega_{00} = 0{\cdot}7$. The figure shows that the results are in excellent agreement. Tables 2 and 3 compare the results well with those of Laura *et al.* [17, 19].

TABLE 3

Comparison of amplitude of dimensionless displacement and radial bending moment at the centre as a function of η ($= \Omega < \Omega_{00}$) and ($= \Omega_{00} < \Omega < \Omega_{01}$) for uniform isotropic plates

		$\Omega < \Omega_{00}$				$\Omega_{00} < \Omega < \Omega_{01}$			
		$W/(a^4 q_0/D_{r0})$		$M_r/a^2 q_0$		$W/(a^4 q_0/D_{r0})$		$M_r/a^2 q_0$	
	η	Reference [19]	Present	Reference [19]	Present	Reference [19]	Present	Reference [19]	Present
$K_\phi = 10^{20} \cong \infty$,									
$\Omega_{00} =$	0·2	0·01630	0·01630	0·085	0·08524	0·011	0·01181	0·084	0·08468
10·2158	0·4	0·01872	0·01872	0·099	0·09952	0·0055	0·00553	0·051	0·05121
$\Omega_{01} =$	0·6	0·02479	0·02479	0·135	0·13533	0·0040	0·00404	0·039	0·03840
39·771	0·8	0·04461	0·04461	0·253	0·25281	0·0044	0·00446	0·080	0·08001
$K_\phi = 0$	0·2	0·06640	0·06640	0·215	0·21554	0·0226	0·02263	0·094	0·09407
$\Omega_{00} =$	0·4	0·07603	0·07603	0·248	0·24874	0·0094	0·00946	0·052	0·05224
4·9351	0·6	0·10010	0·10010	0·331	0·33179	0·0062	0·006238	0·049	0·04948
$\Omega_{01} =$	0·8	0·17875	0·17877	0·603	0·60332	0·0063	0·006329	0·076	0·07653
29·721									

REFERENCES

1. S. G. LEKHNITSKII 1985 *Anisotropic Plates*. New York: Breach Science Publishers.
2. D. G. GORMAN 1983 *Earthquake Engineering and Structural Dynamics* **11**, 843–855. Vibration of thermally stressed polar orthotropic annular plates.
3. D. G. GORMAN 1985 *Journal of Sound and Vibration* **101**, 325–336. Thermal gradient effects upon the vibration of certain composites circular plates, Part I.
4. C. S. KIM and S. M. DICKINSON 1990 *Journal of Sound and Vibration* **143**, 171–179. The flexural vibration of thin isotropic and polar orthotropic annular and circular plates with elastically restrained peripheries.
5. U. S. GUPTA, R. LAL and C. P. VERMA 1986 *Journal of Sound and Vibration* **109**, 423–434. Buckling and vibration of polar orthotropic annular plates on an elastic foundation subjected to hydrostatic peripheral loading.
6. U. S. GUPTA, R. LAL and S. K. JAIN 1991 *Journal of Sound and Vibration* **147**, 423–434. Buckling and vibration of polar orthotropic circular plate of linearly varying thickness resting on elastic foundation.
7. U. S. GUPTA and A. H. ANSARI 1998 *Journal of Sound and Vibration* **213**, 429–445. Free vibration of polar orthotropic circular plates of variable thickness with elastically restrained edge.
8. U. S. GUPTA and A. H. ANSARI 1999 *Indian Journal of Pure and Applied Mathematics* **30**, 975–990. Effect of elastic foundation on asymmetric vibration of polar orthotropic parabolically tapered circular plates.
9. U. S. GUPTA and A. H. ANSARI 1998 *Journal of Sound and Vibration* **215**, 123–150. Asymmetric vibration and elastic stability of polar orthotropic circular plate of linearly varying profile.
10. H. REISMANN 1959 *Journal of Applied Mechanics* **26**, 526. Forced vibration of circular plates.
11. H. REISMANN 1968 *Journal of Applied Mechanics* **35**, 510. Forced vibration of elastic plates.
12. A. J. MCLEOD and R. E. D. BISHOP 1965 *The Forced Vibration of Circular Flat Plates*. Mechanical Engineering Science Monograph, Vol. 1.
13. B. K. DONALDSON 1973 *Journal of Sound and Vibration* **30**, 397–417. A new approach to the forced vibration of thin plates.
14. P. A. A. LAURA and R. DURAN 1975 *Journal of Sound and Vibration* **42**, 129. A note on forced vibrations of a clamped rectangular plate.
15. P. A. A. LAURA, L. E. LUISONI and J. J. LOPEZ 1976 *Journal of Sound and Vibration* **47**, 287–291. A notes on free and forced vibrations of circular plates: the effect of support flexibility.
16. P. A. A. LAURA, J. L. POMBO and L. E. LUISONI 1976 *Journal of Sound and Vibration* **45**, 225–235. Forced vibration of a circular plate elastically restrained against rotation.
17. P. A. A. LAURA, C. FILIPICH and R. D. SANTOS 1977 *Journal of Sound and Vibration* **52**, 243–251. Static and dynamic behaviour of circular plates of variable thickness elastically restrained along the edge.
18. P. A. A. LAURA, B. V. DE GRECO, J. C. UTJES and R. CARNICER 1988 *Journal of Sound and Vibration* **120**, 587. Numerical experiments on free and forced vibrations of beams of non-uniform cross sections.
19. P. A. A. LAURA, D. R. AVALOS and H. A. LARRONDO 1990 *Journal of Sound and Vibration* **136**, 146–150. Forced vibration of circular, stepped plate.
20. K. CHANDRASEKHERAN and V. X. KUNUKKASSERIL 1979 *Journal of Sound and Vibration* **44**, 407. Forced axisymmetric response of circular plate.
21. P. B. BEAUDAN and H. REISMANN 1987 *Journal of Applied Mechanics* **54**, 476. Forced flexural vibration of plates.
22. A. W. LEISSA 1969 *NASA SP*-160, U.S. Washington. Vibration of plates.
23. A. W. LEISSA 1978 *Journal of Sound and Vibration* **56**, 313. A direct method for analyzing the forced vibration of continuous system having damping.
24. A. W. LEISSA 1989 *Journal of Sound and Vibration* **134**, 435. Closed form exact solutions for the steady state vibrations of continuous systems to distributed excited forces.
25. A. W. LEISSA and Y.-T. CHERN 1992 *Journal of Vibration and Acoustics* **114**, 106. Approximate analysis of the forced vibration response of plates.
26. G. N. WEISENSEL and A. L. SCHLACK 1990 *Journal of Analytical and Experimental Model Analysis* **5**, 239. Annular plate response to cirumferentially moving loads with sudden radial position changes.
27. G. N. WEISENSEL and A. L. SCHLACK 1993 *Journal of Applied Mechanics* **60**, 649. Response of annular plates to cirumferentially and radially moving loads.

28. A. P. GUPTA and N. GOEL 1994 *International Journal of Mechanical Sciences* **36**, 439–448. Forced axisymmetric response of linearly tapered circular plates.
29. A. P. GUPTA and N. GOEL 1999 *International Journal of Mechanical Sciences* **220**, 641–657. Forced asymmetric response of linearly tapered circular plates.
30. K. M. LIEW 1992 *Applied Mechanics, ASCE* **118**, 1783–1789. Response of plates of arbitrary shape subjected to static loading.
31. B. V. DE GRECO and P. A. A. LAURA 1984 *Journal of Sound and Vibration* **94**, 525. A note on the forced vibrations of a circular plate with thickness varying in a bilinear fashion.
32. D. V. BAMBILL, P. A. A. LAURA and E. ROMANELLI 1987 *Ocean Engineering* **14**, 527. Forced vibration of a beam elastically restrained against rotation at the ends in the case of a concentrated load.
33. C. FILIPICH, P. A. A. LAURA and R. D. SANTOS 1977 *Journal of Sound and Vibration* **50**, 445. A note on the vibrations of rectangular plates of variable thickness with two opposite simply supported edges and very general boundary conditions on the other two.
34. R. S. WEINER 1965 *Journal of Applied Mechanics* **32**, 893. Forced axisymmetric vibration of circular elastic plates.
35. P. K. ROY and N. GANESAN 1992 *Computers and Structures* **45**, 593. Studies on the response of a square plate with variable thickness excited by uniformly distributed load.
36. S. P. TIMOSHENKO and S. WOINOWSKY-KRIEGER 1959 *Theory of Plates and Shells*. New York, NY: McGraw-Hill.
37. W. NOWACKI 1960 *Dynamics of Elastic Systems*. New York: John Wiley and Sons.